Math 270 Day 7 Part 1

Section 2.6: Substitutions and Transformations (continued)

Section 2.6: Substitutions and Transformations

What we'll go over in this section

- The Substitution/Transformation to use to solve a Homogeneous Equation
- The Substitution/Transformation to use to solve the Equation $\frac{dy}{dx} = G(ax + by)$
- **The Substitution/Transformation to use to solve a Bernoulli Equation**
- **The Substitution/Transformation to use to solve Equations with Linear Coefficients**

Section 2.6: Substitutions and Transformations

<u>The Substitution/Transformation to use to solve a Bernoulli Equation</u>

Bernoulli Equation

Definition 5. A first-order equation that can be written in the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n , \qquad (9)$$

where $P(x)$ and $Q(x)$ are continuous on an interval (a, b) and n is a real number, is called a **Bernoulli equation.**[†]

To solve a Bernoulli Equation

1) Divide both sides of the equation by y^n
2) Make the substitution $v = y^{1-n}$
3) You'll end up with a first-order linear DE (if n is not 0 or 1)

Section 2.6: Substitutions and Transformations

The Substitution/Transformation to use to solve a Bernoulli Equation

Example 3 Solve $\dfrac{dy}{dx} - 5y = -\dfrac{5}{2}xy^3$.

Section 2.6: Substitutions and Transformations

<u>The Substitution/Transformation to use to solve Equations with Linear Coefficients</u>

Equations with Linear Coefficients

$$(a_1x + b_1y + c_1)\,dx + (a_2x + b_2y + c_2)\,dy = 0$$

What does that mean?

- If $b_1 = a_2$, the equation is exact
- If $a_1b_2 = a_2b_1$, the equation can be written in the form $\frac{dy}{dx} = G(ax + by)$
- We'll assume $b_1 \neq a_2$ and $a_1b_2 \neq a_2b_1$
- If $c_1 = c_2$, the equation is homogeneous
- Otherwise, make the substitution $x = u + h$ and $y = v + k$
- Solve for h and k that will make the constants c_1 and c_2 disappear (such a solution exists as long as $a_1b_2 \neq a_2b_1$)
- The equation will now be homogeneous $\frac{dv}{du} = G(v/u)$

Section 2.6: Substitutions and Transformations

The Substitution/Transformation to use to solve Equations with Linear Coefficients

Example 4 Solve $(2x - 2y - 6)dx + (x - 3y - 5)dy = 0$.